Math 2412 Instructor: Adnan Said Chapter 1 Handout

1.2 Basics of Functions and Their Graphs

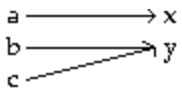
Relation: A relation is a set of ordered pairs.

Function: A function is a relation in which each value of the first component of the ordered pairs corresponds to only one value of the second component.

Relations and functions can be defined in several ways:
1. As a correspondence or mapping.
2. As a set of ordered pairs.
3. As a table.
4. As a graph.
5. As an equation or rule.

Domain and Range: The domain of a relation is the set of all values of the first component of the ordered pairs. The range of a relation is the set of all values of the second component of the ordered pairs.

Decide whether the relation is a function. Give the domain and range.

1) 

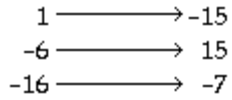
1) _____

The relation is a function.
D: {a, b, c} R: {x, y}

Determine whether the relation is a function. Give the domain and range.

2) {(1, −15), (−6, 15), (−16, −7)}

2) _____



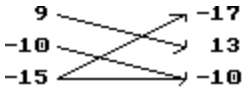
The relation is a function.
D: {1, −6, −16} R: {−15, 15, −7}

Decide whether the relation is a function. Give the domain and range.

3)

x	9	−10	−15	−15
y	13	−10	−17	−10

3) _____



The relation is not a function.

D:$\{9, -10, -15\}$ R:$\{-17, 13, -10\}$

Determine whether the equation represents y as a function of x.

4) $y^2 + x = 4$ 4) _____

Let $x = 0 \rightarrow y^2 = 4 \rightarrow y = \pm\sqrt{4} = \pm 2$

So, $x = 0$ corresponds to two y values.

This means that the equation does not represent a function.

5) $y = |2 - x|$ 5) _____

x	0	1	2	−1	−2	3
y	2	1	0	3	4	1

So, each x value corresponds to only y value.

This means that the equation represents a function.

Function Notation: If y is a function of x, we write y = f(x). Some times letters other than f , such as g or h are used to name functions.

Evaluating Functions: To evaluate a function, replace the variable in the equation of the function with the value given.

Evaluate the function.

6) Let $f(x) = 3x^2 + 5x - 6$. 6) _____

a) Find: f(−4).

$f(-4) = 3(-4)^2 + 5(-4) - 6 = 48 - 20 - 6 = 22$

b) Find: f(m − 2).

$f(m - 2) = 3(m - 2)^2 + 5(m - 2) - 6$

$= 3(m^2 - 4m + 4) + 5(m - 2) - 6$

$= 3m^2 - 12m + 12 + 5m - 10 - 6$

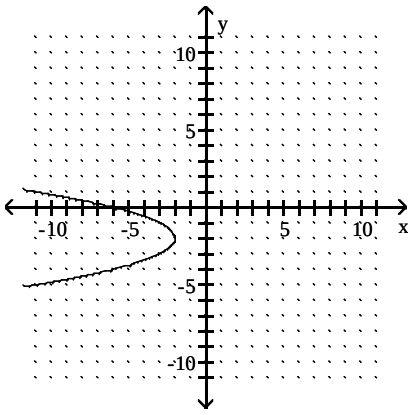
$= 3m^2 - 7m - 4$

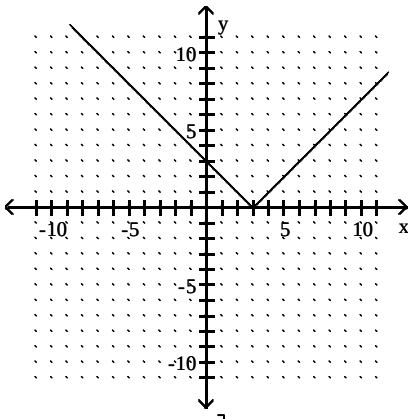
Relations and functions can also be defined as a graph. To distinguish the graph of a function from that of a relation, we use the vertical line test.

Vertical Line Test: If every vertical line intersects the graph of a relation in no more than one point, then the relation is a function.

Decide whether the relation is a function, and give the domain and range.

7) a) b) 7) _____





a) The relation is not a function. D: $(-\infty, -2]$ R: $(-\infty, \infty)$

b) The relation is a function. D: $(-\infty, \infty)$ R: $[0, \infty)$

1.2 Exercises pg 168
Practice: (3, 15, 29, 62 , 71, 79)
More Practice: (5, 21, 31, 63 ,75 ,81)

1.3 More on Functions and Their Graphs

Increasing, Decreasing, and Constant Functions

a. A function f(x) increases on an interval if $x_1 < x_2$ implies $f(x_1) < f(x_2)$.

b. A function f(x) decreases on an interval if $x_1 < x_2$ implies $f(x_1) > f(x_2)$.

c. A function f(x) is constant on an interval if for every x_1 and x_2, $f(x_1) = f(x_2)$.

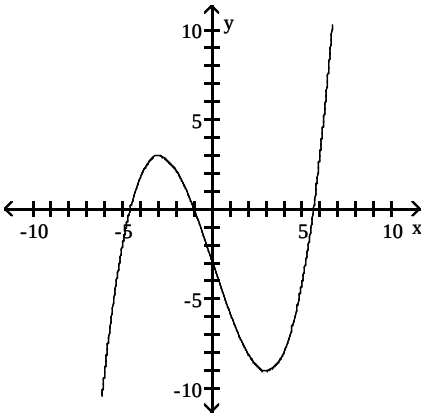
Relative Maxima and Relative Minima

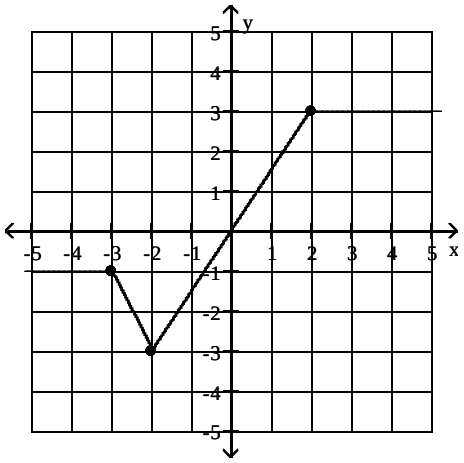
a. If f(x) changes from increasing to decreasing at x = c, then there is a relative maximum at x = c.

b. If f(x) changes from decreasing to increasing at x = c, then there is a relative minimum at x = c.

Determine the intervals over which the function is decreasing, increasing, and constant. Locate the values at which the function has a relative maximum or a relative minimum.

8) a) b) 8) _____





a) The function is increasing over the interval $(-\infty, -3)$.
The function is decreasing over the interval $(-3, 3)$.
The function is increasing over the interval $(3, \infty)$.
f(x) has a relative maximum at x = −3, and
a relative minimum at x = 3.

b) The function is constant over the interval $(-\infty, -3)$.
The function is decreasing over the interval $(-3, -2)$.
The function is increasing over the interval $(-2, 2)$.
The function is constant over the interval $(2, \infty)$.
f(x) has a relative minimum at x = −2, and
no relative maximum .

Even Function:
A function is even if $f(-x) = f(x)$; or if it has symmetry with respect to the y−axis.
Odd Function:
A function is odd if $f(-x) = -f(x)$; or if it has symmetry with respect to the origin.

Decide whether the function is even, odd, or neither.

9) A. $f(x) = x^2 - 4$ 9) _____

Replace x with −x.

→ $f(-x) = (-x)^2 - 4 = x^2 - 4 = f(x)$.

Therefore, the function is even.

B. $f(x) = 4x^5 + 6x^3$

Replace x with −x .

→ $f(-x) = 4(-x)^5 + 6(-x)^3$

$= -4x^5 - 6x^3 = -(4x^5 + 6x^3) = -f(x)$.

Therefore, the function is odd.

Piecewise−Defined Function: A piecewise−defined function is defined by different rules over different intervals of its domain.

Find the requested value.

10) $f(x) = \begin{cases} 4x - 3, & \text{if } x \le -2 \\ 2 - x^2, & \text{if } x > -2 \end{cases}$ 10) _____

$f(-4) = 4(-4) - 3 = -16 - 3 = -19$

$f(-1) = 2 - (-1)^2 = 2 - 1 = 1$

$f(-2) = 4(-2) - 3 = -8 - 3 = -11$

Graph the function.

11) $f(x) = \begin{cases} 5 - x, & \text{if } x \le 2 \\ 2x + 1, & \text{if } x > 2 \end{cases}$ 11) _____

$y = 5 - x$

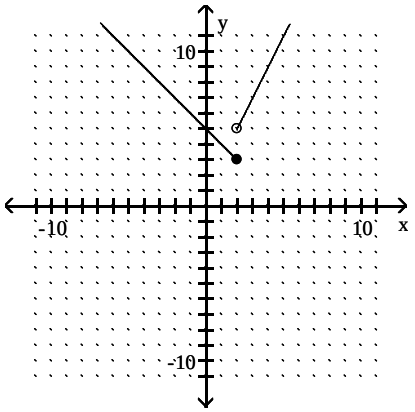
x	y
2	3
0	5

(Full)

$y = 2x + 1$

x	y
2	5
3	7

(Empty)



12) $f(x) = \begin{cases} -x - 1, & x \le 2 \\ -2x + 1, & x > 2 \end{cases}$ 12) _____

$y = -x - 1$

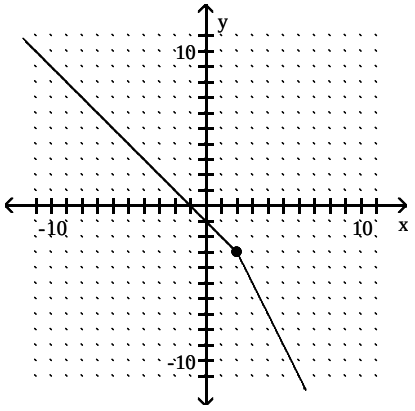
x	y
2	−3
0	−1

(Full)

$y = -2x + 1$

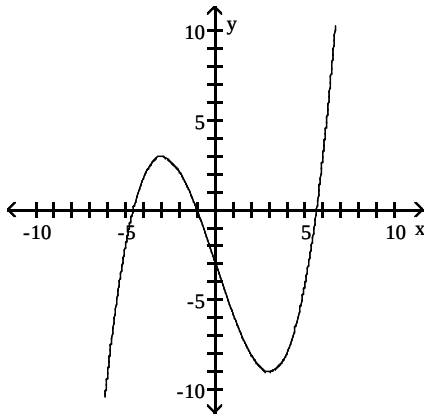
x	y
2	−3
3	−5

(Empty)



Use the graph of f(x) to determine the following:

13) 13) _____



a. the domain of f is D: $(-\infty, \infty)$

b. the range of f is R: $(-\infty, \infty)$

c. $f(3) = -9$

d. a value of x for which $f(x) = -3$ is $x = 0$.

1.3 Exercises pg 182
Practice: (9, 19, 29, 33, 37, 51, 57, 67)
More Practice: (10, 26, 31, 34, 38, 47, 58, 68)

1.4 and 1.5 Linear Functions and Slope

Linear Equation in Two Variables: A linear equation in two variables can be written in the form Ax + By = C, (A and B are not both zero). The graph of any linear equation is the set of points corresponding to all ordered pairs that satisfy the equation. The points lie on a **straight line**.

The Slope Formula: The slope(steepness) of the line passing through the points A(x_1 , y_1) and B(x_2, y_2) is: $m = \frac{\text{Change in y}}{\text{Change in x}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Rise}}{\text{Run}}$.

A positive slope indicates that the line slants up (rises) from left to right.

A negative slope indicates that the line slants down (falls) from left to right.

The x-intercept is the point where the line intersects the x-axis.
To find the x-intercept, set y = 0 and find the x value.

The y-intercept is the point where the line intersects the y-axis.
To find the y-intercept, set x = 0 and find the y value.

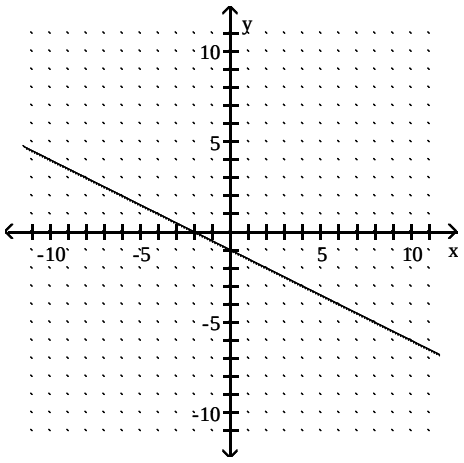
Find the x-intercept and the y-intercept of the line. Sketch the line.

14) $x + 2y = -2$ 14) _____

x	y
−2	0
0	−1

.

The x-intercept is: (−2, 0). The y-intercept is: (0, −1).



Standard Form of the Equation of a Line:

The standard form of the equation of a line is: $Ax + By = C$, where A, B, and C are integers, and $A \geq 0$.

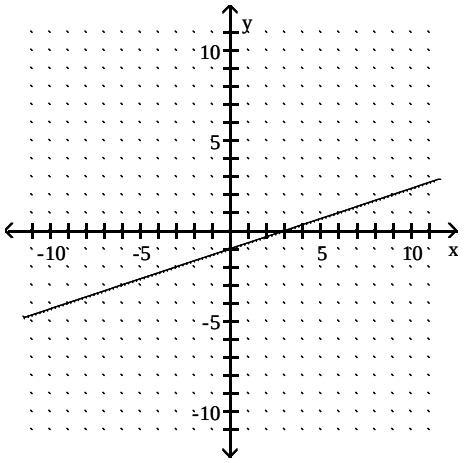
Slope–Intercept Form of the Equation of a Line:

The slope–intercept form of the equation of a line is: $y = mx + b$, with slope m and y–intercept b.

Find the slope and the y–intercept of the line. Sketch the line.

15) $x - 3y = 3 \quad \rightarrow \quad -3y = -x + 3 \quad \rightarrow \quad y = \frac{1}{3}x - 1$ 15) _____

The slope is: $m = \frac{1}{3}$ and the y–intercept is: $b = -1$



Point–Slope Form of the Equation of a Line:

The point–slope form of the equation of a line with slope m passing through the point (x_1 , y_1) is : $y - y_1 = m\,(x - x_1)$

Write the equation of the line. Write the answer in standard form.

16) Through $(-4 , 7)$; $m = \frac{3}{5}$ 16) _____

$y - y_1 = m\,(x - x_1) \quad \rightarrow \quad y - 7 = \frac{3}{5}(x + 4)$

$\rightarrow \; y - 7 = \frac{3}{5}x + \frac{12}{5} \; \rightarrow \quad 5\left[y - 7 = \frac{3}{5}x + \frac{12}{5}\right]$

$\rightarrow \; 5y - 35 = 3x + 12 \; \rightarrow \; -3x + 5y = 47$

$\rightarrow \; -1(-3x + 5y = 47) \; \rightarrow \; 3x - 5y = -47$

Write the equation of the line. Write the answer in slope–intercept form.

17) Through (−2 , −6) and (5 , −8). 17) _____

$$m = \frac{-8-(-6)}{5-(-2)} = \frac{-8+6}{5+2} = -\frac{2}{7}$$

$$y - y_1 = m(x - x_1) \rightarrow y + 6 = -\frac{2}{7}(x + 2)$$

$$\rightarrow y + 6 = -\frac{2}{7}x - \frac{4}{7} \rightarrow y = -\frac{2}{7}x - \frac{4}{7} - 6 \rightarrow y = -\frac{2}{7}x - \frac{46}{7}$$

Parallel Lines: Two lines are parallel if they have the same slope.

Perpendicular Lines: Two lines are perpendicular if the slopes are negative reciprocal of each other.

Find an equation of the line satisfying the conditions. Write the equation in slope–intercept form.

18) Through (9, 2) ; perpendicular to L_2: $3x + 2y = 31$. 18) _____

$$3x + 2y = 31 \rightarrow 2y = -3x + 31 \rightarrow y = -\frac{3}{2}x + \frac{31}{2}$$

The slope of L_2 is : $m = -\frac{3}{2}$ → The slope of line L_1 is : $m = \frac{2}{3}$.

$$y - y_1 = m(x - x_1)$$

$$\rightarrow y - 2 = \frac{2}{3}(x - 9) \rightarrow y - 2 = \frac{2}{3}x - 6$$

$$\rightarrow y = \frac{2}{3}x - 6 + 2 \rightarrow y = \frac{2}{3}x - 4$$

1.4 Exercises pg 199
Practice: (23, 31, 43, 67) More Practice: (24, 38, 45 68)
1.5 Exercises pg 211
Practice: (5, 11) More Practice: (7, 9)

1.6 Transformations of Functions

The most commonly used basic functions in algebra are:

$f(x) = x^2$ (The Quadratic Function)

$f(x) = |x|$ (The Absolute value Function)

$f(x) = \sqrt{x}$ (The Square Root Function)

Graphing techniques presented in this section show how to graph functions by altering the equation of the basic function.

1. Vertical and Horizontal Shifts
   a. To graph $h(x) = f(x) + k$, shift the graph of $f(x)$ k units up.
   b. To graph $h(x) = f(x) - k$, shift the graph of $f(x)$ k units down.
   c. To graph $h(x) = f(x + k)$, shift the graph of $f(x)$ k units to the left.
   d. To graph $h(x) = f(x - k)$, shift the graph of $f(x)$ k units to the right.

2. Stretching and Shrinking
   a. To graph $h(x) = c\, f(x)$, stretch the graph of $f(x)$ vertically if $c > 1$.
   b. To graph $h(x) = c\, f(x)$, shrink the graph of $f(x)$ vertically if $0 < c < 1$.

3. Reflections
   a. To graph $h(x) = -f(x)$, reflect the graph of $f(x)$ across the x− axis.
   b. To graph $h(x) = f(-x)$, reflect the graph of $f(x)$ across the y−axis.

Write an equation for the function that is described by the given characteristics.

19) The shape of $f(x) = x^2$, but shifted 4 units to the right, stretched vertically, shifted 2 units upward, reflected across the x−axis, and a point (3, −1) lying on the graph. 19) _____

$y = -a(x - 4)^2 + 2$

$-1 = -a(3 - 4)^2 + 2 \rightarrow -1 = -a + 2 \rightarrow a = 3$

So, $y = -3(x - 4)^2 + 2$

20) The shape of $f(x) = |x|$, but shifted 3 units to the left, shrunk vertically, shifted 4 units down, reflected across the x−axis, and a point (1, −6) lying on the graph. 20) _____

$y = -a|x + 3| - 4$

$-6 = -a|4| - 4 \quad \rightarrow \quad -6 = -4a - 4 \rightarrow -2 = -4a \rightarrow a = \frac{1}{2}$

So, $y = -\frac{1}{2}|x + 3| - 4$

21) The shape of $f(x) = \sqrt{x}$, but stretched vertically, shifted 5 units upward, reflected across the y−axis, and a point (−4, 11) lying on the graph. 21) _____

$y = a\sqrt{-x} + 5$

$11 = a\sqrt{4} + 5 \rightarrow 11 = 2a + 5 \rightarrow 6 = 2a \rightarrow a = 3$

So, $y = 3\sqrt{-x} + 5$

1.6 Exercises pg 227
Practice: (64, 79, 93, 146, 147)
More Practice: (65, 72, 85, 145)

1.7 Combinations of Functions; Composite Functions

Guidelines in Finding the Domain of a Function

If a function is defined by an equation, use the following guidelines when finding its domain.

1. If the function has a radical with an even index, the domain can be obtained by setting the expression inside the radical to ≥ 0 and solving for x.
2. If the function has a denominator, the domain can be obtained by excluding the values that make the denominator equal to zero.

Give the domain of each function.

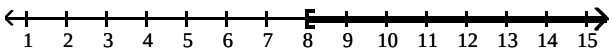
22) $f(x) = x^2 + 3$ 22) _____

x	0	1	2	3	−1	−2	−3
y	3	4	7	12	4	7	12

The domain is all real numbers, also written as $(-\infty , \infty)$.

23) $f(x) = \sqrt{2x - 16}$ 23) _____

Set $2x - 16 \geq 0 \rightarrow 2x \geq 16 \rightarrow x \geq 8$



The domain is : $[8 , \infty)$.

24) a. $g(x) = \dfrac{8}{-3x + 6}$ 24) _____

Set $-3x + 6 = 0 \rightarrow -3x = -6 \rightarrow x = 2$

The domain is all real numbers except $x = 2$, also written as $(-\infty , 2) \cup (2 , \infty)$.

b. $g(x) = \dfrac{7}{6x^2 - 14x}$

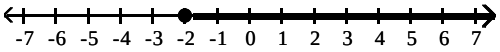
Set $6x^2 - 14x = 0 \rightarrow 2x(3x - 7) = 0$

$2x = 0 \rightarrow x = 0$; $3x - 7 = 0 \rightarrow x = \dfrac{7}{3}$

The domain is all real numbers except $x = 0$ and $x = \dfrac{7}{3}$,

also written as $(-\infty , 0) \cup (0 , \dfrac{7}{3}) \cup (\dfrac{7}{3} , \infty)$.

25) $h(x) = \dfrac{\sqrt{x+2}}{2x-6}$ 25) _____

1. Set $x + 2 \geq 0 \rightarrow x \geq -2$ 

2. Set $2x - 6 = 0 \rightarrow 2x = 6 \rightarrow x = 3$
   Then $x = 3$ has to be excluded from the above interval.

So, the domain is $[-2, \infty)$ excluding $x = 3$, also written as:
$[-2, 3) \cup (3, \infty)$.

Combinations of Functions: Given two functions f(x) and g(x).

The Sum: $(f + g)(x) = f(x) + g(x)$

The Difference: $(f - g)(x) = f(x) - g(x)$

The Product: $(fg)(x) = f(x) \cdot g(x)$

The Quotient: $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}$, $g(x) \neq 0$

Domains

The domains of $(f + g)(x)$, $(f - g)(x)$, $(fg)(x)$, and $\left(\dfrac{f}{g}\right)(x)$ include all real numbers in the intersection of the domains of f(x) and g(x).

In the case of the quotient $\left(\dfrac{f}{g}\right)(x)$, there is the further restriction that $g(x) \neq 0$.

Perform the requested operation.

26) $f(x) = 2x - 3$, $g(x) = x^2 - 4x$ 26) _____

a. $(f + g)(x)$ (Find the domain)

$(f + g)(x) = (2x - 3) + (x^2 - 4x) = x^2 - 2x - 3$ D: $(-\infty, \infty)$.

b. $(f - g)(x)$ (Find the domain)

$(f - g)(x) = (2x - 3) - (x^2 - 4x) = 2x - 3 - x^2 + 4x$

$= -x^2 + 6x - 3$ D: $(-\infty, \infty)$.

c. $(f + g)(3) = (3)^2 - 2(3) - 3 = 9 - 6 - 3 = 0$

d. $(f - g)(-2) = -(-2)^2 + 6(-2) - 3 = -4 - 12 - 3 = -19$

27) $f(x) = 2x - 3$, $g(x) = x^2 - 4x$ 27) _____

a. $(fg)(x)$ (Find the domain)

$(fg)(x) = (2x - 3)(x^2 - 4x) = 2x^3 - 8x^2 - 3x^2 + 12x$

$= 2x^3 - 11x^2 + 12x$ D: $(-\infty , \infty)$.

b. $(\frac{f}{g})(x)$ (Find the domain)

$$(\frac{f}{g})(x) = \frac{2x - 3}{x^2 - 4x}$$

Set $x^2 - 4x = 0 \rightarrow x(x - 4) = 0$

$\rightarrow x = 0$; $x - 4 = 0 \rightarrow x = 4$

The domain is all real numbers except $x = 0$ and $x = 4$, also written as $(-\infty , 0) \cup (0 , 4) \cup (4, \infty)$.

c. $(fg)(-4) = 2(-4)^3 - 11(-4)^2 + 12(-4)$

$= -128 - 176 - 48 = -352$

d. $$(\frac{f}{g})(-6) = \frac{2(-6) - 3}{(-6)^2 - 4(-6)} = \frac{-12 - 3}{36 + 24} = \frac{-15}{60} = -\frac{1}{4}$$

The Composition of Functions: The composition of the function f with the function g is denoted $f \circ g$ and is defined as: $(f \circ g)(x) = f(g(x))$

Domains: The domain of $(f \circ g)(x)$ must not include the values that are not in the domain of g(x).

Perform the requested operation.

28) $f(x) = x^2 + 7x$, $g(x) = x - 6$ 28) _____

a. $(f \circ g)(x)$ (Find the domain)

$(f \circ g)(x) = f(g(x)) = f(x - 6) = (x - 6)^2 + 7(x - 6)$

$= x^2 - 12x + 36 + 7x - 42 = x^2 - 5x - 6$ D: $(-\infty , \infty)$.

b. $(g \circ f)(x)$ (Find the domain)

$(g \circ f)(x) = g(f(x)) = g(x^2 + 7x) = x^2 + 7x - 6$ D: $(-\infty , \infty)$.

c. $(f \circ g)(-1) = (-1)^2 - 5(-1) - 6 = 1 + 5 - 6 = 0$

d. $(g \circ f)(-2) = (-2)^2 + 7(-2) - 6 = 4 - 14 - 6 = -16$

29) $f(x) = \frac{2}{x - 1}$, $g(x) = \frac{3}{x}$ 29) _____

a. $(f \circ g)(x)$ (Find the domain)

$$(f \circ g)(x) = f(g(x)) = f(\frac{3}{x}) = \frac{2}{\frac{3}{x} - 1} = \frac{2x}{3 - x}$$

$x = 0$ is not in the domain of g(x), it can not be in the domain of $(f \circ g)(x)$.

Also, $x = 3$ can not be in the domain of $(f \circ g)(x)$

So, the domain of $(f \circ g)(x)$ is all real numbers except $x = 0$ and $x = 3$, also written as $(-\infty , 0) \cup (0 , 3) \cup (3, \infty)$.

b. $(g \circ f)(x)$ (Find the domain)

$$(g \circ f)(x) = g(f(x)) = g(\frac{2}{x - 1}) = \frac{3}{\frac{2}{x - 1}} = 3 \cdot \frac{x - 1}{2} = \frac{3(x - 1)}{2}$$

$x = 1$ is not in the domain of f(x), it can not be in the

domain of $(g \circ f)(x)$.
So, the domain of $(g \circ f)(x)$ is all real numbers except $x = 1$,
also written as $(-\infty, 1) \cup (1, \infty)$.

c. $(f \circ g)(1) = \frac{2}{3-1} = \frac{2}{2} = 1$

d. $(g \circ f)(-2) = \frac{3(-2-1)}{2} = \frac{-9}{2}$

1.7 Exercises pg 242
Practice: (7, 28, 35, 55, 69)
More Practice: (23, 25, 37, 54, 73)

1.8 Inverse Functions

One-to-One Function:
A function is one-to-one if no two x-values have the same y-value .

Determine whether or not the function is one-to-one.

30) $f(x) = x^4 + 3$ 30) _____

x	-1	1
y	4	4

The function is not one-to-one.

31) $f(x) = 2x - 7$ 31) _____

x	0	1	2	-1	-2	3
y	-7	-5	-3	-9	-11	-1

The function is one-to-one.

32) $f(x) = x^3 - 4$ 32) _____

x	0	1	2	-1	-2	3
y	-4	-3	4	-5	-12	23

The function is one-to-one.

33) $f(x) = \dfrac{x + 1}{x - 2}$ 33) _____

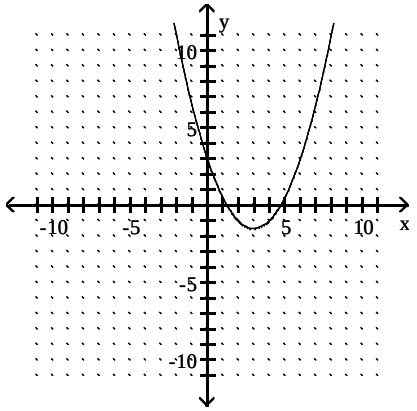
x	0	1	3	−1	−2	4
y	− 1/2	−2	4	0	1/4	5/2

The function is one−to−one.

Horizontal Line Test: A function is one−to−one if any horizontal line intersects the graph of a function at only one point.

Determine whether or not the function is one−to−one.

34) 34) _____

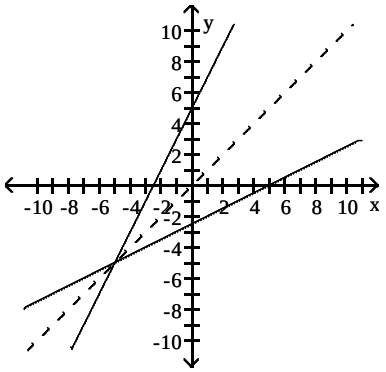


The function is not one−to−one.

Inverse Functions: Two functions are inverses of each other if their graphs are reflections of each other across the line y = x.

Decide whether or not the functions are inverses of each other.

35) 35) _____



The functions are inverses of each other.

Finding the Inverse of a Function:

If a function is one-to-one, then the inverse exists.

1. Exchange x and y. 2. Solve for y. 3. Replace y with $f^{-1}(x)$.

The domain of f(x) is the same as the range of $f^{-1}(x)$, and the domain of $f^{-1}(x)$ is the same as the range of f(x).

If f is one-to-one, find an equation for its inverse.

36) $f(x) = x^4 + 3$ 36) _____

The function is not one-to-one, so the inverse does not exist.

37) $f(x) = 2x - 7$ 37) _____

Step1: $x = 2y - 7$ Step2: $y = \frac{x+7}{2}$ Step3: $f^{-1}(x) = \frac{x+7}{2}$

38) $f(x) = x^3 - 4$ 38) _____

Step1: $x = y^3 - 4$

Step2: $y^3 = x + 4 \rightarrow y = \sqrt[3]{x+4}$

Step3: $f^{-1}(x) = \sqrt[3]{x+4}$

39) $f(x) = \frac{x+1}{x-2}$ 39) _____

Step1: $x = \frac{y+1}{y-2}$

Step2: $x(y-2) = y + 1 \rightarrow xy - 2x = y + 1$

$\rightarrow xy - y = 2x + 1 \rightarrow y(x-1) = 2x + 1$

$\rightarrow y = \frac{2x+1}{x-1}$

Step3: $f^{-1}(x) = \frac{2x+1}{x-1}$

1.8 Exercises pg 254

Practice: (19, 27, 31, 37, 52)

More Practice: (15, 17, 34, 36)

1.9 Distance and Midpoint Formulas; Circles

Distance Formula: The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by : $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Find the distance between the pair of points.

40) $(-1, 4)$ and $(3, -2)$ 40) _____

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(3 + 1)^2 + (-2 - 4)^2}$$
$$= \sqrt{(4)^2 + (-6)^2} = \sqrt{16 + 36} = \sqrt{52} = \sqrt{4 \cdot 13} = 2\sqrt{13}$$

Midpoint Formula

The midpoint of the line segment with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ is:

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Find the midpoint of the line segment joining the two points.

41) $(1, -6)$ and $(-8, -4)$ 41) _____

$$\frac{x_1 + x_2}{2} = \frac{1 + (-8)}{2} = \frac{-7}{2}$$

$$\frac{y_1 + y_2}{2} = \frac{-6 + (-4)}{2} = \frac{-10}{2} = -5$$

So, the midpoint is: $M\left(\frac{-7}{2}, -5\right)$.

Standard Form of the Equation of a Circle

The circle with center (h, k) and radius r has equation: $(x - h)^2 + (y - k)^2 = r^2$

Write the standard form of the equation of a circle.

42) center $(-3, 5)$, radius $\sqrt{3}$ 42) _____

$h = -3$, $k = 5$, $r = \sqrt{3}$

$(x - h)^2 + (y - k)^2 = r^2$

→ The equation is: $(x + 3)^2 + (y - 5)^2 = 3$

Find the center and radius of the circle.

43) $(x + 4)^2 + y^2 = 8$ 43) _____

$h = -4$; $k = 0$ The center is: $(-4, 0)$

$r^2 = 8 \rightarrow r = \sqrt{8} = 2\sqrt{2}$ The radius is: $r = 2\sqrt{2}$ units

Write the standard form of the equation of a circle.

44) The point (3, 4) lies on a circle whose center is $(-1, 2)$ 44) _____

The radius is the distance between the point on the circle and the center.

$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-1 - 3)^2 + (2 - 4)^2}$

$= \sqrt{(-4)^2 + (-2)^2} = \sqrt{16 + 4} = \sqrt{20}$

$(x - h)^2 + (y - k)^2 = r^2$

$h = -1$, $k = 2$, $r = \sqrt{20}$

The equation is: $(x + 1)^2 + (y - 2)^2 = 20$

The General Form of the Equation of a Circle

$x^2 + y^2 + Dx + Ey + F = 0$, where D, E, and F are real numbers.

Find the center and radius of the circle .

45) $x^2 + y^2 + 8x - 6y + 16 = 0$ 45) _____

1. Rearrange the terms so that x-terms and y-terms are arranged in descending order, and the constant term appears on the right.
   $x^2 + 8x + y^2 - 6y = -16$
2. Complete the square on x and y.
   $x^2 + 8x + 16 + y^2 - 6y + 9 = -16 + 16 + 9$
   $(x + 4)^2 + (y - 3)^2 = 9$
   $h = -4$, $k = 3$ So, the center is: $(-4, 3)$.
   $r^2 = 9 \rightarrow r = \sqrt{9} = 3$

1.9 Exercises pg 264

Practice: (17, 25, 37, 43, 53) More Practice: (7, 23, 39, 51, 60)